

Dynamic Instability of Finned Missiles Caused by Unbalanced Fin Forces

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A dynamic instability is described for rolling, finned missiles with canted fins. The instability occurs as an undamping of the angle of attack and is caused by differential lift and drag from unequal effectiveness of the windward and leeward fins. The instability is similar to a Magnus instability in that a yawing moment is produced that unequally damps the positive and negative precession motions. The equivalence of the Magnus coefficient and the differential fin forces is derived, and the instability is demonstrated with computer simulations of the equations of motion for the angle-of-attack convergence of a re-entry vehicle. The computer results are compared with a closed-form solution for the angle-of-attack convergence envelope.

Nomenclature

A	= slope of fin lift effectiveness vs angle of attack
B	= slope of fin drag effectiveness vs angle of attack
C_{l_0}	= aerodynamic fin-induced roll moment coefficient
$C_{l\delta}$	= aerodynamic fin lift derivative
$C_{m\dot{q}}$	= aerodynamic pitch damping derivative
$C_{n\dot{p}\alpha}$	= aerodynamic Magnus moment derivative
$C_{N\alpha}$	= aerodynamic normal force derivative
$C'_{N\alpha}$	= $2C_{N\alpha}I/md^2$
d	= aerodynamic reference diameter
D	= fin drag force
I	= pitch or yaw moment of inertia
I_x	= roll moment of inertia
L	= fin lift force
m	= vehicle mass
$M_{\dot{\xi}}$	= aerodynamic yaw moment
$M_{\dot{\eta}}$	= aerodynamic pitch moment
$M_{\dot{\xi}}$	= aerodynamic roll moment
p	= roll rate
p_r	= reduced roll rate, $\mu p/2$
q	= dynamic pressure
S	= aerodynamic reference area
u	= vehicle velocity
x_f	= distance of fins aft of center of mass
x_{st}	= static margin (distance of center of pressure aft of center of mass)
x, y, z	= body-fixed coordinates
X, Y, Z	= inertial coordinates
δ	= effective fin cant angle
δ_0	= fixed fin cant angle relative to body axis of symmetry
ζ	= yaw axis
η	= pitch axis
θ	= total angle of attack (Euler angle)
κ	= roll moment coefficient, $C_{l_0}qSd/I_x$
λ	= fin drag coefficient, $C_{D\delta}qSd/2I$
μ	= inertia ratio, I_x/I
ν	= pitch or yaw damping coefficient, $(qSd^2/2Iu)(-C_{m\dot{q}} + 2C_{N\alpha}I/md^2)$
ν_m	= $(\mu C_{N\alpha}qS/mu) + (\delta_0\tau A - \lambda)/p$
ξ	= roll axis
σ	= $2\omega/\mu p$
Σ	= $[\sigma^2 + (\tau + \lambda B\delta_0)/p_r]^2$
τ	= fin lift coefficient, $C_{l\delta}qSx_f/I$
φ	= roll orientation relative to wind (Euler angle)
ψ	= precession angle (Euler angle)
ω	= natural pitch frequency

Introduction

It has been observed in flight and demonstrated mathematically that a statically stable missile at zero roll rate can become dynamically unstable at sufficiently high roll rates as a result of Magnus forces.¹⁻³ The Magnus forces induce positive damping that, for sufficiently high roll rates, overcomes the negative yaw damping that is usually present to some degree. Magnus forces are characteristically dependent on both angle of attack and roll rate, analogous to the classical Magnus lift on a spinning cylinder in a cross flow. As such, Magnus instabilities have been observed on both finned and unfinned bodies of revolution.

The addition of fins to an axisymmetric missile can alter its stability characteristics by increasing the classical Magnus forces or by causing Magnus-type effects that have a different origin than the classical Magnus forces.⁴⁻⁶ It has been demonstrated from wind-tunnel tests^{4,5} that body interference on a finned missile at angle of attack can cause a fin lift unbalance that produces a net yawing moment analogous to the body-induced Magnus moment. The fin-induced yaw moment can act in the same or opposite direction to the body-induced Magnus moment, depending on the angle of attack and on whether or not the missile has reached its equilibrium roll rate dictated by the fin cant angle and freestream velocity. A similar effect is caused by unequal fin drag forces when the vehicle angle of attack causes a difference in the effective fin cant angles of opposite fins in a plane perpendicular to the angle-of-attack plane.⁶

This paper describes a mathematical model for predicting Magnus-type instabilities having origins similar to those identified in Refs. 4-6. The present analysis differs from those cited in that the fin dimensions are assumed to be small relative to the body so that span-wise variations in fin forces can be neglected. This is more characteristic of re-entry vehicles as compared with missiles that operate at lower Mach numbers. Roll damping can also be neglected in this case since the effective fin cant due to roll is small for re-entry roll rates of interest. Consequently, the mechanism proposed herein for the fin-lift-induced Magnus-type instability actually differs from that described by Platou^{4,5} due to body-fin interference and results in a Magnus moment of opposite sign. For hypersonic flow with canted fins, the roll rate, in general, remains small relative to the equilibrium free spin roll rate. Hence the fin lift forces that oppose roll (roll damping) remain small relative to the lift forces that drive the roll rate. At angle of attack the windward fins have increased effectiveness and therefore produce a net yaw force that acts in the same direction as the classical Magnus force on a spinning

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cylinder. The Magnus-type moment described by Benton⁶ due to differential fin drag from canted fins at angle of attack is also included here and acts in the opposite direction to the classical Magnus moment.

A closed-form solution is obtained for the angle-of-attack convergence envelope of a rolling re-entry vehicle with canted spin fins. This is an extension of previous work, in which a solution was obtained for the angle-of-attack convergence envelope of a nonfinned re-entry vehicle with roll acceleration and pitch/yaw damping.^{7,8}

Euler Angle Coordinates

The vehicle rotational motion is described in terms of the Euler angles ψ, ϕ, θ , which describe the position of a set of body-fixed axes x, y, z , relative to an inertial frame X, Y, Z that translates with the vehicle, as shown in Fig. 1. The axes ξ, η, ζ are axes of roll, pitch, and yaw, respectively, relative to the plane of total angle of attack. They precess about the velocity vector with angular rate $\dot{\psi}$. The Euler angle θ is the total angle of attack, and the angle ϕ is the roll angle relative to the wind. The roll rate p is then the roll rate relative to the wind $\dot{\phi}$ plus the component of precession $\dot{\psi} \cos \theta$ along the roll axis; i.e.,

$$p = \dot{\phi} + \dot{\psi} \cos \theta \quad (1)$$

If the principal moments of inertia about the ξ, η, ζ axes are I_x, I_y, I_z , respectively, and the aerodynamic moments about these axes are M_ξ, M_η, M_ζ , the moment equations of motion in terms of the Euler angles for an axisymmetric vehicle may be written⁹

$$M_\xi = I_x \ddot{\phi} \quad (2a)$$

$$M_\eta = I \ddot{\theta} + I_x p \dot{\psi} \sin \theta - I \dot{\psi}^2 \sin \theta \cos \theta \quad (2b)$$

$$M_\zeta = I d/dt(\dot{\psi} \sin \theta) + I \dot{\theta} \dot{\psi} \cos \theta - I_x p \dot{\theta} \quad (2c)$$

Aerodynamic Moments

It is assumed that the only forces acting on the vehicle that contribute to the aerodynamic moments M_ξ, M_η, M_ζ are normal forces from angle of attack, fin lift and drag forces from canted spin fins, and pitch and yaw damping forces. Roll damping is neglected for reasons previously cited. For simplicity, only two spin fins are considered, although the results so obtained could be generalized to any number. The fins are assumed to be located at the aft end of the vehicle a distance x_f behind the vehicle center of mass, as shown in Fig. 2. The fins are canted to produce a positive (clockwise) roll moment and are designated Fin 1 and Fin 2 in order to differentiate between the windward and leeward positions at any instant. The roll angle ϕ describes the roll orientation of the vehicle relative to the plane of total angle of attack and is

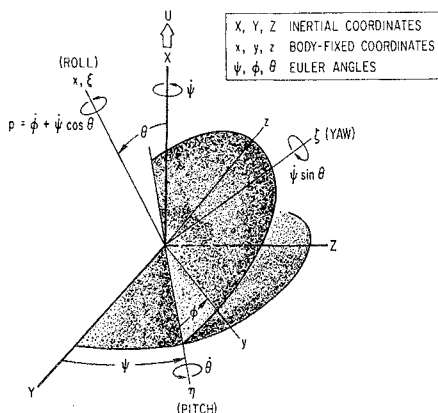


Fig. 1 Euler angle coordinates.

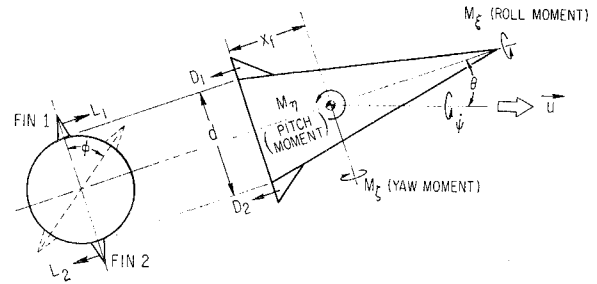


Fig. 2 Vehicle configuration.

referenced such that $\phi = 0$ when Fin 1 is leeward and Fin 2 is windward. The aerodynamic moments due to the fin lift forces L_1 and L_2 and drag forces D_1 and D_2 are then,

$$M_{\xi_{fins}} = (L_1 + L_2)d/2 \quad (3a)$$

$$M_{\eta_{fins}} = (L_1 - L_2)x_f \sin \phi + (D_1 - D_2)(d/2) \cos \phi \quad (3b)$$

$$M_{\zeta_{fins}} = (L_2 - L_1)x_f \cos \phi + (D_1 - D_2)(d/2) \sin \phi \quad (3c)$$

and the total aerodynamic moments acting on the vehicle are

$$M_\xi = M_{\xi_{fins}} \quad (4a)$$

$$M_\eta = M_{\eta_{fins}} - C_{N_\alpha} \theta q S x_{ci} - (q S d^2/2u) (-C_{m_q} + C'_{N_\alpha}) \dot{\theta} \quad (4b)$$

$$M_\zeta = M_{\zeta_{fins}} - (q S d^2/2u) \times [(-C_{m_q} + C'_{N_\alpha}) \dot{\psi} \sin \theta - \mu C'_{N_\alpha} p \theta] \quad (4c)$$

The last terms in the second and third of Eqs. (4) are pitch and yaw damping moments, where $\dot{\theta}$ and $\dot{\psi} \sin \theta$ are the pitch and yaw rates, respectively, and $C'_{N_\alpha} = 2C_{N_\alpha} I / m d^2$.

The fin lift and drag forces, both assumed to be proportional to fin cant, are given by

$$L_1 = C_{l_\delta^{(1)}} \delta_1 q S \quad (5a)$$

$$L_2 = C_{l_\delta^{(2)}} \delta_2 q S \quad (5b)$$

$$D_1 = C_{D_\delta^{(1)}} \delta_1 q S \quad (5c)$$

$$D_2 = C_{D_\delta^{(2)}} \delta_2 q S \quad (5d)$$

where C_{l_δ} and C_{D_δ} are fin lift and drag derivatives, respectively, and δ_1 and δ_2 are the effective cant angles of Fins 1 and 2, respectively. The effective cant angles consist of the fixed cant angle δ_o of the fins relative to the axis of symmetry of the vehicle plus the component of angle of attack that may add to or subtract from δ_o , depending on the roll orientation ϕ . The effective cant angles can be written

$$\delta_1 = \delta_o - \theta \sin \phi \quad (6a)$$

$$\delta_2 = \delta_o + \theta \sin \phi \quad (6b)$$

The dependence of the fin lift and drag forces on angle of attack is expressed through the fin lift and drag derivatives according to

$$C_{l_\delta^{(1)}} = C_{l_\delta} [1 + \theta f_1(\phi)] \quad (7a)$$

$$C_{l_\delta^{(2)}} = C_{l_\delta} [1 + \theta f_2(\phi)] \quad (7b)$$

$$C_{D_\delta^{(1)}} = C_{D_\delta} [1 + \theta g_1(\phi)] \quad (7c)$$

$$C_{D_\delta^{(2)}} = C_{D_\delta} [1 + \theta g_2(\phi)] \quad (7d)$$

where C_{l_δ} and C_{D_δ} are the zero-angle-of-attack values and the $f(\phi)$ and $g(\phi)$ are periodic functions that describe the windward and leeward fin positions. The simplest first-order linear dependence on angle of attack is assumed. If we choose for

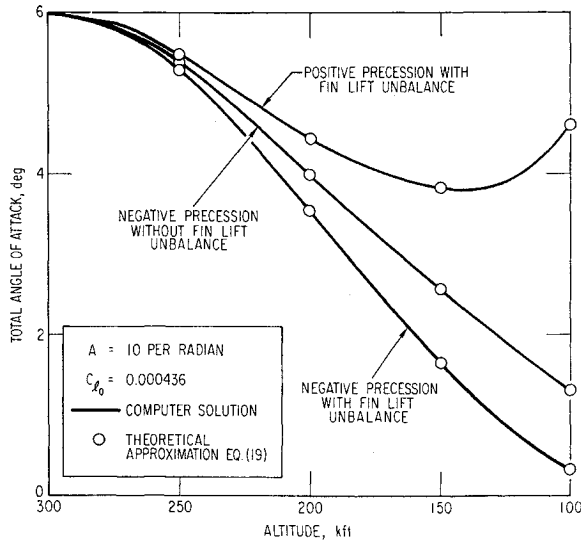


Fig. 3 Comparison of angle-of-attack histories.

the $f(\varphi)$ and $g(\varphi)$ functions,

$$f_2(\varphi) = -f_1(\varphi) = A \cos \varphi \quad (8)$$

$$g_2(\varphi) = -g_1(\varphi) = B \cos \varphi$$

then the windward and leeward values of $C_{l\delta}$ become

$$(C_{l\delta})_{\text{windward}} = C_{l\delta}(1 + A\theta) \quad (9a)$$

$$(C_{l\delta})_{\text{leeward}} = C_{l\delta}(1 - A\theta) \quad (9b)$$

with similar expressions for $C_{D\delta}$. For this truly linear case, cross-flow effects are equally large on leeward and windward sides. Equations (5-8) give, for the fin-induced moments of Eq. (3), the expressions

$$M_{\xi_{\text{fins}}} = C_{l\delta} q S d (\delta_0 + A\theta^2 \sin \varphi \cos \varphi) \quad (10a)$$

$$M_{\eta_{\text{fins}}} = -2C_{l\delta} q S x_f \theta (A\delta_0 \sin \varphi \cos \varphi + \sin^2 \varphi) - C_{D\delta} q S d \theta (B\delta_0 \cos^2 \varphi + \sin \varphi \cos \varphi) \quad (10b)$$

$$M_{\zeta_{\text{fins}}} = 2C_{l\delta} q S x_f \theta (A\delta_0 \cos^2 \varphi + \sin \varphi \cos \varphi) - C_{D\delta} q S d \theta (B\delta_0 \sin \varphi \cos \varphi + \sin^2 \varphi) \quad (10c)$$

For angles of attack greater than the trim values from configuration and aerodynamic asymmetries (assumed to be zero here), the windward-meridian rotation rate $\dot{\varphi}$ will, in general, be nonzero⁷; i.e., the vehicle rolls about its axis relative to the wind. Consequently, the sinusoidal functions of φ in Eq. (10) can be averaged over a cycle to yield average values for the aerodynamic moments due to the unbalanced fin forces. Making use of the definite integrals,

$$\frac{1}{2\pi} \int_0^{2\pi} \sin \varphi \cos \varphi d\varphi = 0$$

$$\frac{1}{2\pi} \int_0^{2\pi} \sin^2 \varphi d\varphi = \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \varphi d\varphi = \frac{1}{2}$$

we obtain for the average values of these moments,

$$\bar{M}_{\xi_{\text{fins}}} = C_{l\delta} \delta_0 q S d \quad (11a)$$

$$\bar{M}_{\eta_{\text{fins}}} = C_{l\delta} q S x_f \theta - C_{D\delta} q S d \theta \delta_0 B/2 \quad (11b)$$

$$\bar{M}_{\zeta_{\text{fins}}} = C_{l\delta} q S x_f \theta \delta_0 A - C_{D\delta} q S d \theta/2 \quad (11c)$$

Equations of Motion

The moment equations of motion in terms of the Euler angle coordinates, obtained from substitution of the aero-

dynamic moments from Eqs. (4) and (11) into Eq. (2), can be written

$$\begin{aligned} \kappa \dot{\theta} &= \dot{p} \\ -(\omega^2 + \tau + \lambda B \delta_0) \theta - \nu \dot{\theta} &= \ddot{\theta} + \mu p \dot{\psi} \sin \theta - \dot{\psi}^2 \sin \theta \cos \theta \\ -\nu \dot{\psi} \sin \theta + \nu_m p \dot{\theta} &= \frac{d}{dt} (\dot{\psi} \sin \theta) + \dot{\theta} \dot{\psi} \cos \theta - \mu p \dot{\theta} \end{aligned} \quad (12)$$

where the coefficients are

$$\kappa = C_{l\delta} \delta_0 q S d / I_x \quad (13a)$$

$$\tau = C_{l\delta} q S x_f / I \quad (13b)$$

$$\nu = (q S d^2 / 2 I u) (-C_{mq} + C'_{N\alpha}) \quad (13c)$$

$$\lambda = C_{D\delta} q S d / 2 I \quad (13d)$$

$$\omega^2 = C_{N\alpha} q S x_f / I \quad (13e)$$

$$\mu = I_x / I \quad (13f)$$

$$\nu_m = (\mu C_{N\alpha} q S / m u) + (\delta_0 \tau A - \lambda) / p \quad (13g)$$

The parameter ω is the natural pitch frequency of the vehicle.

Equivalent Magnus Coefficient and Yaw Damping

The yaw moment $\bar{M}_{\zeta_{\text{fins}}}$ in Eq. (11), created by the fin force unbalance, is equivalent to a Magnus moment:

$$M_{\text{magnus}} = C_{np\alpha} (q S d^2 / 2 u) p \theta \quad (14)$$

in which the Magnus coefficient $C_{np\alpha}$ has the value,

$$C_{np\alpha} = [A C_{l\delta} \delta_0 (x_f / d) - C_{D\delta} / 2] (2u / p d) \quad (15)$$

The lift and drag components act in opposite directions, so that either positive or negative Magnus effects are possible. Unlike the classical Magnus moment, the yaw moment $\bar{M}_{\zeta_{\text{fins}}}$ is independent of roll rate in the first-order approximation. This follows from the assumption of negligible roll damping. The fin lift derivative $C_{l\delta}$ is defined such that the product $C_{l\delta} \delta_0$ is the zero-angle-of-attack value of the total roll moment coefficient $C_{l\delta}$ induced by the canted spin fins.

The fin-induced yaw moment can be expressed as an equivalent yaw damping coefficient. From the yaw equation of motion in Eqs. (12), with the small-angle approximation $\sin \theta \approx \theta$, the fin unbalance term is equivalent to a yaw damping coefficient ν_{fins} of magnitude

$$\nu_{\text{fins}} = (\lambda - \delta_0 \tau A) / \dot{\psi} \quad (16)$$

Following the approach of Ref. 7, we find the quasi-steady value of the precession rate $\dot{\psi}$ from the pitch equation of motion. Again using the small angle approximations $\sin \theta \approx \theta$ and $\cos \theta \approx 1$, we can write this equation in the form

$$\ddot{\theta} + (\omega^2 + \tau + \lambda B \delta_0 + \mu p \dot{\psi} - \dot{\psi}^2) \theta + \nu \dot{\theta} = 0 \quad (17)$$

which yields for the two precession modes the quasi-steady values

$$\bar{\dot{\psi}}_{+,-} = p_r [1 \pm (1 + \Sigma^2)^{1/2}] \quad (18)$$

where

$$\Sigma^2 = (\omega^2 + \tau + \lambda B \delta_0) / p_r^2 \quad (19)$$

The term $\tau + \lambda B \delta_0$ arises from the fin force unbalance and increases slightly the static stability of the vehicle. Since Eq. (18) yields one positive and one negative value for the two precession modes, the equivalent yaw damping from Eq. (16) will be either positive or negative, depending on which mode prevails and on the relative magnitude of the fin lift and drag components.

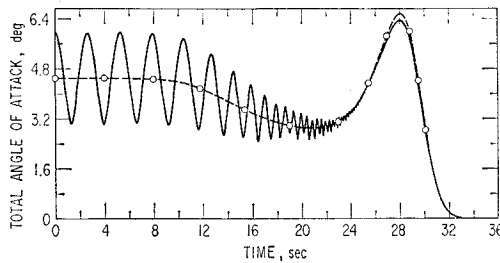


Fig. 4 Angle-of-attack behavior for velocity vector inside exoatmospheric precession cone.

Re-Entry Vehicle Angle-of-Attack Convergence Envelope

The influence of the fin lift unbalance on the angle-of-attack convergence of a rolling re-entry vehicle can be determined directly from the results of Refs. 7 and 8. The angle-of-attack convergence envelope, from Ref. 8 with $\Sigma^2 = \sigma^2 + (\tau + \lambda B\delta_o)/p_r^2$ substituted for σ^2 , can be written

$$\frac{\bar{\theta}}{\theta_o} = (1 + \Sigma^2)^{-1/4} \exp \left\{ -\frac{1}{2} \int_0^t \left\{ \frac{\dot{p}}{p} + \nu \pm \frac{[(\dot{p}/p) + \nu - (2/\mu)\nu_m]}{(1 + \Sigma^2)^{1/2}} \right\} dt' \right\} \quad (20)$$

The top sign in the \pm sign in the exponent corresponds to the positive precession mode, and the bottom sign corresponds to the negative precession mode. The parameter σ is the ratio of the natural pitch frequency to the reduced roll rate p_r . For a slender vehicle with $\mu = 10$, for example, $\sigma^2 = 400/(p/\omega)^2$, which is a strong function of the ratio p/ω . For roll rates not excessively supercritical, $\sigma^2 \gg 1$ and $\Sigma^2 + 1 \approx \Sigma^2$. With the further assumption that $(\tau + \lambda B\delta_o) \ll \omega^2$, Eq. (20) reduces to the simpler form,

$$\frac{\bar{\theta}}{\theta_o} = \left(\frac{\mu p_o}{2\omega} \right)^{1/2} \exp \left\{ -\frac{1}{2} \int_0^t \left[\nu \pm \frac{p_r}{\omega} \left(\frac{\dot{p}}{p} + \nu - \frac{2}{\mu} \nu_m \right) \right] dt' \right\} \quad (21)$$

For conditions such that undamping due to fin force unbalance predominates, Eq. (21) simplifies even further to

$$\bar{\theta}/\theta_o = (\mu p_o/2\omega)^{1/2} \exp \left\{ -1/2 \int_0^t [\nu \mp (\delta_o \tau A - \lambda)/\omega] dt' \right\} \quad (22)$$

To illustrate the effect of unbalanced fin forces, we compare the result, Eq. (20), with digital computer solutions of the equations of motion, Eq. (12). The unbalanced fin drag forces are assumed to be small relative to the lift forces and are neglected in the following examples. Figure 3 shows a comparison of angle-of-attack histories for the two extreme cases of exoatmospheric motion that result in nearly circular motion in both the positive and negative precession modes.⁷ Also shown, for comparison, is the angle-of-attack behavior for a negative precession case in which unbalanced fin forces are nonexistent. The theoretical approximation, Eq. (20), is included for comparison in each case. For re-entries in which the axis of the exoatmospheric precession cone does not coincide with the velocity vector, the initial angle of attack will be oscillatory. The initial average value θ_o of the oscillation envelope for such cases is determined by the angle between the velocity vector and the axis of the precession cone. If the velocity vector lies inside the precession cone, θ_o is equal to the precession cone half angle. If the velocity vector lies outside the precession cone, then θ_o is the angle between the velocity vector and the axis of the precession cone. Figures 4 and 5 show computer simulations of two such cases in which the

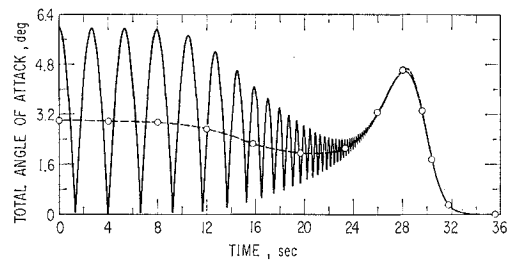


Fig. 5 Angle-of-attack behavior for velocity vector tangent to exoatmospheric precession cone.

velocity vectors lie inside the exoatmospheric precession cones. For the case shown in Fig. 5, the velocity vector is almost tangent to the precession cone but lies just inside such that the positive precession mode prevails. The theoretical approximations to the average values of the oscillation envelopes, computed from Eq. (20), are shown for comparison.

Summary and Conclusions

A Magnus-type instability of finned missiles, caused by unbalanced fin lift and drag forces from unequal effectiveness of opposite fins, has been described quantitatively. Although of different origin than the classical Magnus moment of a spinning cylinder in a cross flow, the fin-induced yaw moment can act in the same or opposite direction, depending on the relative magnitudes of the fin lift and drag forces. Unlike the classical Magnus moment, the fin-induced yaw moment is independent of roll rate in the first-order linear approximation. The equivalent Magnus coefficient is found to have the value

$$C_{n\alpha} = (AC_{l\alpha}x_f/d - C_{D\delta}/2)(2u/pd)$$

where A is the slope of the fin lift effectiveness vs angle-of-attack curve, assumed to be linear, $C_{l\alpha}$ is the zero-angle-of-attack fin-induced roll moment coefficient, and $C_{D\delta}$ is the fin drag force coefficient per unit cant angle. A closed-form solution employing this result has been obtained for the angle-of-attack convergence envelope of a finned re-entry vehicle. The result indicates the magnitude of the fin lift unbalance required to cause a net undamping of the angle of attack.

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